

## Multiuser Detection over Generalized-K Fading Channels with MRC Receive Diversity in Presence of Impulsive Noise

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### ABSTRACT

In direct sequence-code division multiple accesses systems (DS-CDMA), the signals are transmitted over multipath channels that introduce fading and shadowing. Multipath fading and shadowing along with multiple access interference and inter-symbol interference degrades the system performance. Further, experimental results have confirmed the presence of impulsive noise in wireless mobile communication channels. Hence, this paper presents a technique for multiuser detection in DS-CDMA systems over generalized-K (GK) fading channels in presence of impulsive noise. An approximate closed-form expression for average probability of error of an M-decorrelator is derived, for the demodulation of binary phase shift keying (BPSK) signals transmitted over the channels with simultaneous presence of fading and shadowing. Maximal ratio combining (MRC) receive diversity is also incorporated to mitigate the effects of fading and shadowing. Performance of a new M-estimator based detector is also studied by evaluating error rate with the derived expression. Simulation results reveal that the proposed M-estimator based detector performs better in the presence of fading, shadowing and heavy-tailed impulsive noise.

Keywords: Diversity combining, fading channel; GK distribution; impulsive noise; multiuser detection; probability of error, shadowing.

### I. INTRODUCTION

Recent research has explored the potential benefits of multiuser detection for code division multiple access (CDMA) communication systems with present multiple access interference (MAI) [1]. These optimal multiuser detectors have led to the developments of the various linear multiuser detectors with Gaussian noise though various experimental results have confirmed that many realistic channels are impulsive in nature [2], [3]. Lately, the problem of robust multiuser detection in non-Gaussian channels has been addressed in the literature [4], [5], [6], which were developed based on the Huber, Hampel, and a new M-estimator (modified Hampel), respectively. Since CDMA transmissions are frequently made over channels that exhibit fading, it is of interest to design receivers that take this fading behavior of the channel in to account [3]. Robust multiuser detection for synchronous DS-CDMA system with maximal ratio

combiner (MRC) receive diversity over Nakagami-m fading channels is presented in [7] by assuming that the modulation is binary PSK (BPSK). But, the simultaneous presence of multipath fading and shadowing leads to worsening the wireless channels [8]. Recently, the performance of M-decorrelator in simultaneous presence of fading and shadowing with impulsive noise is presented in [9]. Hence, in this paper, the problem of multiuser detection for DS-CDMA system over generalized-K (GK) fading channels with impulsive noise is considered. D-branch MRC receive diversity is incorporated to mitigate the effects of multipath fading.

DS-CDMA system over multipath fading channel in non-Gaussian impulsive noise environment is presented in Section II. Section III presents the GK fading channel with MRC receive diversity. Influence function of the proposed M-estimator is

presented in Section IV and an approximate closed-form expression for average probability of error of an M-decorrelator is also derived. Section V presents some simulation results. Section VI concludes the paper.

## II. SYSTEM MODEL

An L-user CDMA system, where each user transmits information by modulating a signature sequence over a single-path Nakagami-m fading channel, is considered in this paper. The received signal over one symbol duration can be modeled as [3]

$$r(t) = \Re \left\{ \sum_{l=1}^L \sum_{i=0}^{M-1} b_l(i) \alpha_l(t) e^{j\theta_l(t)} s_l(t - iT_s - \tau_l) \right\} + n(t) \quad (1)$$

where  $\Re\{\cdot\}$  denotes the real part, M is the number of data symbols per user in the data frame of interest,  $T_s$  is the symbol interval,  $\alpha_l(t)$  is the time-varying fading gain of the  $l^{\text{th}}$  user's channel,  $\theta_l(t)$  is the time-varying phase of the  $l^{\text{th}}$  user's channel,  $b_l(i)$  is the  $i^{\text{th}}$  bit of the  $l^{\text{th}}$  user,  $s_l(t)$  is the normalized signaling waveform of the  $l^{\text{th}}$  user and  $n(t)$  is assumed as a zero-mean complex non-Gaussian noise. It is assumed that the signaling constellation consists of non-orthogonal signals given by [3]

$$s_l(t) = \begin{cases} \sqrt{\frac{2}{T}} a_l(t) e^{j(\omega_c t + \theta_l)} & \text{for } t \in [0, T_s] \\ 0 & \text{for } t \notin [0, T_s] \end{cases} \quad (2)$$

where  $j = \sqrt{-1}$ ,  $\theta_l$  is the phase of the  $l^{\text{th}}$  user relative to some reference,  $\omega_c$  is the common carrier frequency, and the spreading waveforms,  $a_l(t)$ , are of the form

$$a_l(t) = \sum_{n=1}^N a_n^l u_{T_c}(t - (n-1)T_c) \quad (3)$$

where  $\{a_n^l\}_{n=1}^N$  is a signature sequence of +1s and -1s assigned to the  $l^{\text{th}}$  user, and  $u_{T_c}(t)$  is a unit-amplitude pulse of duration  $T_c$  with  $T_s = NT_c$ . At the receiver, the received signal  $r(t)$  is first filtered by a chip-matched filter and then sampled at the chip rate,  $1/T_c$ . The resulting discrete-time signal sample corresponding to the  $n^{\text{th}}$  chip of the  $i^{\text{th}}$  symbol is given by [3]

$$r_n(i) = \sqrt{\frac{2}{T_c}} \int_{iT_s + nT_c}^{iT_s + (n+1)T_c} r(t) e^{-j\omega_c t} dt, n = 1 \dots N. \quad (4)$$

For synchronous case (i.e.,  $\tau_1 = \tau_2 = \dots = \tau_L = 0$ ), assuming that the fading process for each user varies at a slower rate that the magnitude and phase can taken to be constant over the duration of a bit, (4) can be expressed in matrix notation as [3]

$$\underline{r}(i) = \underline{A}\underline{\theta}(i) + \underline{w}(i) \quad (5)$$

where

$$\underline{r}(i) \square [\underline{r}_1(i), \dots, \underline{r}_N(i)]^T \quad (6)$$

$$\underline{w}(i) \square [\underline{w}_1(i), \dots, \underline{w}_N(i)]^T \quad (7)$$

$$\underline{\theta}(i) \square \frac{1}{\sqrt{N}} [\underline{b}_1(i) \underline{g}_1(i), \dots, \underline{b}_L(i) \underline{g}_L(i)]^T \quad (8)$$

with  $\underline{w}_n(i)$  is a sequence of independent and identically distributed (i.i.d.) complex random variables whose in-phase and quadrature components are independent non-Gaussian random variables with probability density function (PDF) of this noise model has the form

$$f = (1 - \varepsilon) \mathcal{N}(0, \nu^2) + \varepsilon \mathcal{N}(0, \kappa \nu^2) \quad (9)$$

with  $\nu > 0$ ,  $0 \leq \varepsilon \leq 1$ , and  $\kappa \geq 1$ . Here  $\mathcal{N}(0, \nu^2)$  represents the nominal background noise and the  $\mathcal{N}(0, \kappa \nu^2)$  represents an impulsive component, with  $\varepsilon$  representing the probability that impulses occur. It is assumed that the  $l^{\text{th}}$  user employs binary phase shift keying (BPSK) modulation to transmit the data bits  $b_l \in [-1, 1]$  with equal probability and a symbol rate  $1/T_s$ .

## III. GK FADING CHANNEL WITH MRC DIVERSITY

It is assumed that the signal of each user arrives at the receiver through an independent, single-path fading channel. For the shadowed fading channels,  $\alpha_l(i)$  are i.i.d. random variables with GK distribution given by [8]

$$p_\alpha(\alpha_l) = \frac{2}{\Gamma(m)\Gamma(\mu)} \left( \sqrt{\frac{m\mu}{\Omega_0}} \right)^{m+\mu} \alpha_l^{\frac{m+\mu}{2}-1} K_{m-\mu} \left( 2 \sqrt{\frac{m\mu}{\Omega_0}} \alpha_l \right) \quad (10)$$

where,  $m$  is the Nakagami fading parameter that determines the severity of the fading,  $\mu$  represents the shadowing levels,  $\Omega_0$  is the average SNR in a shadowed fading channel,  $K_\xi(\cdot)$  is the modified Bessel function and  $\Gamma(\cdot)$  is the Gamma function [10].

Assuming that the fading is mitigated through D-branch MRC, the output of maximal ratio combiner can be written as [8, 11]

$$R = \sum_{j=1}^D \alpha_j \quad (11)$$

where  $\alpha$  are i.i.d. GK distributed random variables each having a PDF of the form (10). The PDF of  $R$ , with multipath fading and shadowing from branch to branch are distinct, is given by [8]

$$p_R(r) = \frac{2}{\Gamma(m_m)\Gamma(\mu_m)} \left( \sqrt{\frac{m_m\mu_m}{\Omega_0}} \right)^{m_m+\mu_m} \times r^{\frac{m_m+\mu_m-1}{2}} \times K_{m_m-\mu_m} \left( 2\sqrt{\frac{m_m\mu_m}{\Omega_0}} r \right) \quad (12)$$

where

$$m_m = Dm + (D-1) \left[ \frac{-0.127-0.95m-0.0058\mu}{1+0.00124m+0.98\mu} \right] \quad (13)$$

and

$$\mu_m = D\mu. \quad (14)$$

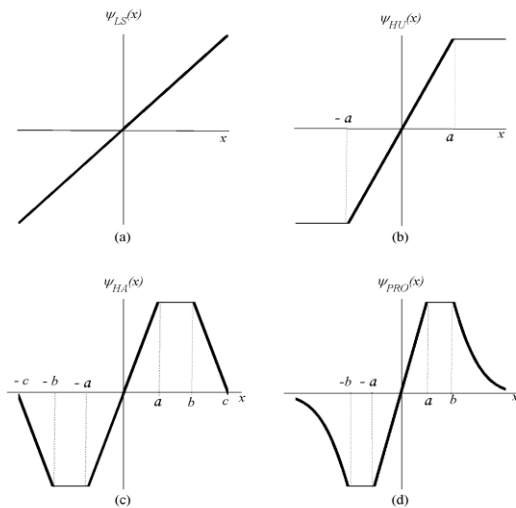


Fig. 1. Influence functions of (a) the linear decorrelating detector, (b) Huber estimator, (c) Hampel estimator, and (d) the proposed estimator.

#### IV. MULTI TAPER SPECTRUM ESTIMATION METHOD

In this section, the proposed M-estimator is presented and the average probability of error is derived for a single-path shadowed fading channel.

##### A) M-estimator

An M-estimator is, a generalization of usual maximum likelihood estimates, used to estimate the unknown parameters  $\theta_1, \theta_2, \dots, \theta_L$  (where  $\theta = Ab$ ) by minimizing a sum of function  $\rho(\cdot)$  of the residuals [4]

$$\hat{\theta} = \arg \min_{\theta \in \mathbb{R}^L} \sum_{j=1}^N \rho \left( r_j - \sum_{l=1}^L s_j^l \theta_l \right) \quad (15)$$

where  $\rho$  is a symmetric, positive-definite function with a unique minimum at zero, and is chosen to be less increasing than square and  $N$  is the processing gain. An influence function  $\psi(\cdot) = \rho'(\cdot)$  is proposed (as shown in Fig. 1 along with other influence functions), such that it yields a solution that is not sensitive to outlying measurements, as [6]

$$\psi_{PRO}(x) = \begin{cases} x & \text{for } |x| \leq a \\ a \operatorname{sgn}(x) & \text{for } a < |x| \leq b \\ \frac{a}{b} x \exp \left( 1 - \frac{x^2}{b^2} \right) & \text{for } |x| > b \end{cases} \quad (16)$$

where the choice of the constants  $a$  and  $b$  depends on the robustness measures.

##### B) Average probability of error

The asymptotic probability of error for the class of decorrelating detectors, for large processing gain  $N$ , is given by [4]

$$P_e^I \equiv \Pr(\hat{\theta}_l < 0 | \theta_l > 0) = Q \left( \frac{A_l}{v \sqrt{[\mathbf{R}^{-1}]_{ll}}} \right) \quad (17)$$

where  $Q(x)$  is the Gaussian Q-function [11],

$$v^2 = \frac{\int \psi^2(u) f(u) du}{\left[ \int \psi'(u) f(u) du \right]^2} \quad (18)$$

and  $\mathbf{R} = \mathbf{S}^T \mathbf{S}$  with  $\mathbf{S}$  is an  $N \times L$  matrix of columns  $\mathbf{a}_l$ . Average probability of error can be obtained by averaging the conditional probability of error (17) over the PDF, (12), of maximal ratio combiner output as

$$\overline{P_e^I} = F \cdot \int_0^\infty x^{\left( \frac{m_m+\mu_m-1}{2} \right)} Q \left( \frac{x}{v \sqrt{[\mathbf{R}^{-1}]_{ll}}} \right) \times K_{m_m-\mu_m} \left( 2\sqrt{\frac{m_m\mu_m}{\Omega_0}} x \right) dx \quad (19)$$

where  $F = \frac{2}{\Gamma(m_m)\Gamma(\mu_m)} \left( \sqrt{\frac{m_m\mu_m}{\Omega_0}} \right)^{m_m+\mu_m}$ . Using the well known upper-bound approximation, called Chernoff bound, to  $Q(x)$ , given by

$$Q(x) \leq \frac{1}{2} e^{-x^2/2} \quad (20)$$

the integral of (19) can be expressed as

$$\int_0^\infty x^{\left(\frac{m_m + \mu_m - 1}{2}\right)} \exp\left(\frac{-x^2}{\nu^2 [\mathbf{R}^{-1}]_{11}}\right) \times K_{m_m - \mu_m} \left(2\sqrt{\frac{m_m \mu_m}{\Omega_o}} x\right) dx \quad (21)$$

Now, by using [Eq. 6.631.3, 10] to evaluate the integral (21), the average probability of error can be derived as [9]

$$\overline{P_e^1} = F \cdot \frac{1}{2} \xi^{-0.5l} \beta^{-1} \Gamma\left(\frac{1+d+l}{2}\right) \Gamma\left(\frac{1-d+l}{2}\right) \cdot \exp\left(\frac{\beta^2}{8\xi}\right) W_{-0.5l,d}\left(\frac{\beta^2}{4\xi}\right) \quad (22)$$

where  $d = m_m - \mu_m$ ,  $l = \frac{m_m + \mu_m}{2} - 1$ ,  $\xi = \frac{1}{\nu^2 [\mathbf{R}^{-1}]_{11}}$

and  $\beta = 2\sqrt{\frac{m_m \mu_m}{\Omega_o}}$  and  $W_{\lambda,\gamma}(\cdot)$  is the Whittaker function [9].

## V. SIMULATION RESULTS

In this section, the performance of M-decorrelator is presented by computing (22) for different values of order of diversity. It is assumed that  $m = 1.4$ ,  $\mu = 2$  and  $\Omega_o = 10$  dB in the simulations.

Performance of M-decorrelator with different influence functions is shown in Fig. 2, Fig. 3 and Fig. 4. In Fig. 2, the average probability of error versus the signal-to-noise ratio (SNR) corresponding to the user 1 under perfect power control of a synchronous DS-CDMA system with six users ( $L = 6$ ) and a processing gain,  $N = 31$  is plotted for Gaussian noise ( $\varepsilon = 0$ ). In Fig. 3, the average probability of error is plotted for moderate impulsive noise ( $\varepsilon = 0.01$ ). Similarly, in Fig. 4, the average probability of error is plotted for highly impulsive noise ( $\varepsilon = 0.1$ ). Simulation results show that the proposed M-estimator based detector performs well compared to linear multiuser detector, minimax detector with Huber and Hampel estimator based detectors.

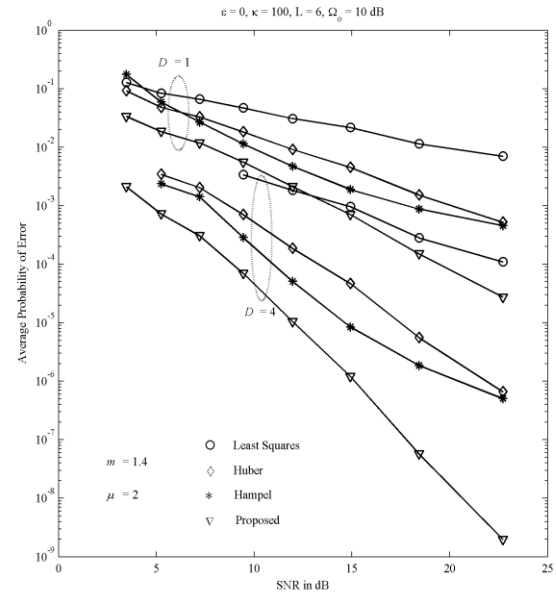


Fig. 2. Average probability of error versus SNR for user 1 for linear multiuser detector (Least Squares), minimax detector with Huber, Hampel and proposed M-estimator in synchronous CDMA channel with impulse noise,  $N = 31$ ,  $\varepsilon = 0$ .

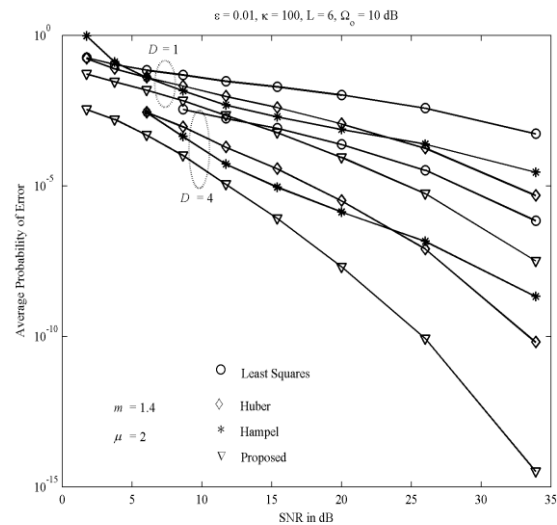


Fig. 3. Average probability of error versus SNR for user 1 for linear multiuser detector (Least Squares), minimax detector with Huber, Hampel and proposed M-estimator in synchronous CDMA channel with impulse noise,  $N = 31$ ,  $\varepsilon = 0.01$ .

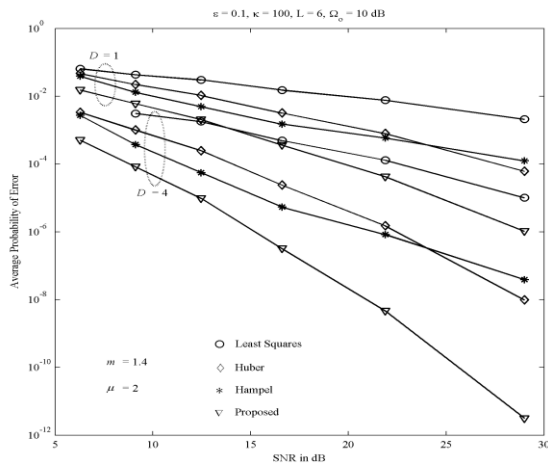


Fig. 4. Average probability of error versus SNR for user 1 for linear multiuser detector (Least Squares), minimax detector with Huber, Hampel and proposed M-estimator in asynchronous CDMA channel with impulse noise,  $N = 31$ ,  $\epsilon = 0.1$ .

## VI. CONCLUDING REMARKS

Multiuser detection for DS-CDMA systems over GK fading channels under impulsive noise environment is presented. An approximate closed-form expression for average probability of error of the M-decorrelator to detect BPSK signals is derived by incorporating MRC receive diversity. An M-estimator based multiuser detector is proposed and its performance is analyzed by computing average probability of error using the expression derived. Simulation results show that the proposed multiuser detector offers significant performance gain over the linear multiuser detector and the minimax decorrelating detector with Huber and Hampel M-estimators, in heavy-tailed impulsive noise.

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